

Complex to Rational Fast Matrix Multiplication

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Key Question

Given a *complex*-coefficient scheme, can we find an *equivalent* one with *rational* coefficients?

Or, more ambitiously, an *integer* equivalent?

Context: AlphaEvolve (Google DeepMind) found fast algorithms over $\mathbb{C}$ and $\mathbb{Q}[i]$.

Dumas, Pernet, Sedoglavic (2025) found rational equivalents via ad-hoc methods and educated guessing.

Our Contribution

Main Result

A more **systematic method** that, given a complex-coefficient MM scheme, either:

- ① Constructs an equivalent **rational algorithm** under some condition, or
- ② **Proves** that no equivalent rational scheme exists.

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- 2 **Proves** that no equivalent rational scheme exists.

Positive results:

- Recovers Dumas, Pernet, Sedoglavic for AlphaEvolve's $\langle 4, 4, 4, 48 \rangle$ and $\langle 3, 4, 7, 63 \rangle$
- Extends to general $\mathbb{F}[\sqrt{d}]$ including $i = \sqrt{-1}$

New non-existence results:

- Smirnov's $\langle 4, 9, 4, 104 \rangle \mathbb{Q}[\sqrt{161}]$ alg. admits **no rational equivalent**
- Handles integer-coefficient non-existence too

Encoding a Fast Matrix Multiplication Algorithm

A fast algorithm multiplying $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$ using t scalar multiplications can be encoded as a **sum of t rank-one tensors**:

$$\mathcal{T} = \sum_{j=1}^t G_j \otimes H_j \otimes K_j, \quad G_j \in \mathbb{R}^{m \times n}, \quad H_j \in \mathbb{R}^{n \times p}, \quad K_j \in \mathbb{R}^{p \times m}.$$

We write $\langle m, n, p, t \rangle$ for such a scheme.

Brent Equations

The decomposition correctly computes AB if and only if for all $i_1, i_2 \in [m]$, $j_1, j_2 \in [n]$, $k_1, k_2 \in [p]$:

$$\sum_{\ell=1}^t G_{\ell}(i_1, j_1) H_{\ell}(j_2, k_1) K_{\ell}(k_2, i_2) = \delta_{i_1 i_2} \delta_{j_1 j_2} \delta_{k_1 k_2}.$$

De-Groote Actions

Given matrix multiplication tensor decomposition $\mathcal{T} = \sum_{j=1}^t G_j \otimes H_j \otimes K_j$ with $G_j \in \mathbb{F}^{m \times n}$, $H_j \in \mathbb{F}^{n \times p}$, $K_j \in \mathbb{F}^{p \times m}$, and invertible X, Y, Z of compatible sizes, the following are called the **De-Groote actions**:

- ① $\sum_{j=1}^t H_j \otimes K_j \otimes G_j$ (**Cyclic permutation**)
- ② $\sum_{j=1}^t H_j^T \otimes G_j^T \otimes K_j^T$ (**Transpose and transposition**)
- ③ $\sum_{j=1}^t (XG_jY^{-1}) \otimes (YH_jZ^{-1}) \otimes (ZK_jX^{-1})$ (**Sandwich action**)

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Key Property

De-Groote actions keep a MM tensor decomposition valid.

Rationalization via De-Groote Actions

Setup: Suppose G_j, H_j, K_j have entries in $\mathbb{F}[\sqrt{d}]$ (e.g. $\mathbb{Q}[i]$).

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Prior Work — Dumas, Pernet, Sedoglavic (2025)

In elegant recent work, they were the first to use De-Groote's sandwich action to *rationalize* complex MM tensors — cleverly exploiting symmetry and well-chosen **heuristics** (educated guesses) to find rational equivalents for AlphaEvolve's $\langle 4, 4, 4, 48 \rangle$ and $\langle 3, 4, 7, 63 \rangle$ algorithms.

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Our work: A **more automated algorithm** that either constructs such X, Y, Z when they exist or proves no rational equivalent exists.

Key Idea

Recall: $\mathbb{F}[\sqrt{d}]$ is the original (complex-like) field, e.g. $\mathbb{Q}[i]$; $\mathbb{F}$ is the target field, e.g. $\mathbb{Q}$.

Rationalization

$$XG_jY^{-1}, YH_jZ^{-1}, ZK_jX^{-1} \in \mathbb{F}$$

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Necessary Condition

$$\begin{aligned} X(G_jH_jK_j)X^{-1} &\in \mathbb{F} \\ Y(H_jK_jG_j)Y^{-1} &\in \mathbb{F} \\ Z(K_jG_jH_j)Z^{-1} &\in \mathbb{F} \end{aligned}$$

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Key insight: Rationalizability reduces to finding X, Y, Z that *simultaneously conjugate* the cyclic products into $\mathbb{F}$ — solvable by linear algebra.

Question: Can we find a **necessary and sufficient condition** for the existence of such X, Y, Z directly from the cyclic products $G_jH_jK_j, H_jK_jG_j, K_jG_jH_j$?

Key Theorem

Let $A \in M_n(\mathbb{F}[\sqrt{d}])$. Consider the two conditions:

Goal

$$\exists X \in GL_n(\mathbb{F}[\sqrt{d}]) \text{ such that} \\ XAX^{-1} \in M_n(\mathbb{F})$$

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($\Rightarrow$) If $XAX^{-1} \in M_n(\mathbb{F})$, take $S = \bar{X}^{-1}X$. **Verification:** since $XAX^{-1} \in \mathbb{F}$ is fixed by conjugation,

$$XAX^{-1} = \bar{X}\bar{A}\bar{X}^{-1} \iff \bar{X}^{-1}XA = \bar{A}\bar{X}^{-1}X \iff SA = \bar{A}S.$$

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In our setting, $A = G_j H_j K_j$, so the condition reads

$$S(G_j H_j K_j) = \overline{G_j H_j K_j} S.$$

From Theorem to Algorithm

In our context: apply the theorem to $A = G_j H_j K_j$ (and cyclic variants) for all j simultaneously.

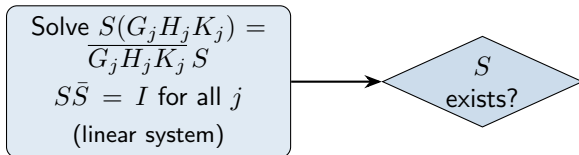
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$$\begin{aligned} \text{Solve } S(G_j H_j K_j) &= \overline{G_j H_j K_j} S \\ S \bar{S} &= I \text{ for all } j \\ &\text{(linear system)} \end{aligned}$$

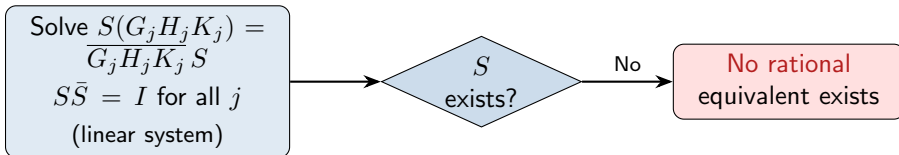
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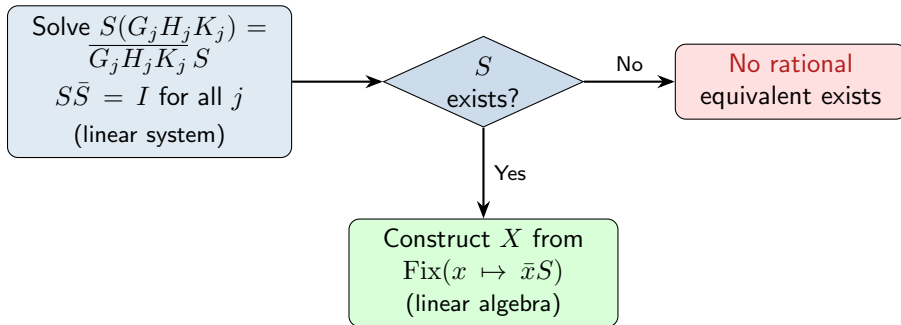
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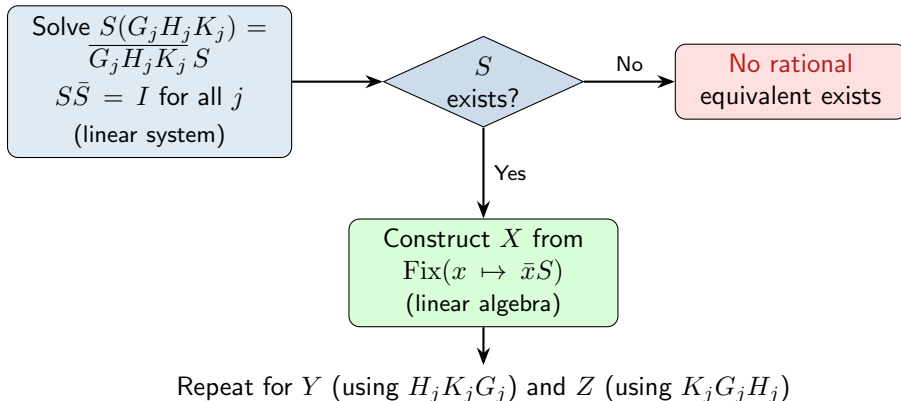
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A Potential Issue

What the algorithm guarantees:

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Counterexample: Take

$$XG_1 Y^{-1} = \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}, \quad YH_1 Z^{-1} = \begin{pmatrix} -i & 0 \\ 0 & 1 \end{pmatrix}, \quad ZK_1 X^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then $X(G_1 H_1 K_1) X^{-1} = I \in \mathbb{F}$, but $XG_1 Y^{-1} \otimes YH_1 Z^{-1} \otimes ZK_1 X^{-1}$ is **not in $\mathbb{F}$** .

Why May It Fail?

Recall: Solve $SM_j = \overline{M_j}S$ and $\bar{S}S = I$ for all $j = 1, \dots, t$ simultaneously.

Root cause: the S we solve for may not be unique.

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Assumption: S is unique up to scalar

We say S is **unique up to scalar** if any two solutions S, S' satisfy $S' = \alpha S$ for some $\alpha \in \mathbb{F}[\sqrt{d}]$ with $\alpha\bar{\alpha} = 1$. Under this assumption, the algorithm always makes the correct output.

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This assumption is mild. Solving $SM_j = \overline{M_j}S$ for all j *simultaneously* means intersecting solution spaces — each equation reduces the dimension, so S is generically unique. **This assumption solves the problem.**

Going Further: Is There an *Integer* Equivalent?

Once a tensor is rationalized over $\mathbb{Q}$, we can ask for more: an equivalent over $\mathbb{Z}$ would be **faster still**. The *same* machinery applies: a De-Groote action conjugates the cyclic products $M_j = G_j H_j K_j \mapsto X M_j X^{-1}$, and conjugation **preserves the trace** of *any* product $M_{j_1} M_{j_2} \cdots M_{j_k}$.

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A trace obstruction

Integer matrices have integer traces. So if *any* product $M_{j_1} \cdots M_{j_k}$ has $\text{Trace}(M_{j_1} \cdots M_{j_k}) \notin \mathbb{Z}$, then **no** X, Y, Z can make the tensor integer.

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In practice: check products of increasing length until one fails. E.g. single traces already rule out AlphaEvolve's $\langle 3, 4, 7, 63 \rangle$; going to pairs and triples rules out every other algorithm in our results table too.

Results: Applying the Algorithm

Tensor	Found by	Original	Best reached	No equiv. over
$\langle 4, 4, 4, 48 \rangle$	AlphaEvolve	$\mathbb{Q}[i]$	$\mathbb{Q}$	$\mathbb{Z}$
$\langle 3, 4, 7, 63 \rangle$	AlphaEvolve	$\mathbb{Q}[i]$	$\mathbb{Q}$	$\mathbb{Z}$
$\langle 4, 9, 4, 104 \rangle$	Smirnov	$\mathbb{Q}[\sqrt{161}]$	$\mathbb{Q}[\sqrt{161}]$	$\mathbb{Q}$
$\langle 3, 3, 6, 40 \rangle$	Smirnov	$\mathbb{Z}[1/2]$	$\mathbb{Z}[1/2]$	$\mathbb{Z}$
$\langle 2, 4, 5, 32 \rangle$	Hopcroft–Kerr	$\mathbb{Z}[1/2]$	$\mathbb{Z}[1/2]$	$\mathbb{Z}$
$\langle 2, 4, 4, 26 \rangle$	Benson–Ballard	$\mathbb{Z}[1/2]$	$\mathbb{Z}[1/2]$	$\mathbb{Z}$

Green: our algorithm finds a strictly simpler ring (the two AlphaEvolve schemes are *rationalized*, $\mathbb{Q}[i] \rightarrow \mathbb{Q}$ — recovering Dumas, Pernet, and Sedoglavic’s results). **No equiv. over:** proven that *no* De-Groote-equivalent exists over that ring (e.g. via the trace obstruction).

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- **A more automated method.** Turns rationalizing a complex-coefficient scheme into a linear-algebra computation.
- **A sharp yes/no answer.** Under a mild uniqueness condition, it either builds a rational equivalent or certifies that none exists.
- **One step further: integers.** A simple trace test tells us, for free, when no integer equivalent can exist either.

Thank you!

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